

Quantum Bruhat

Graphs

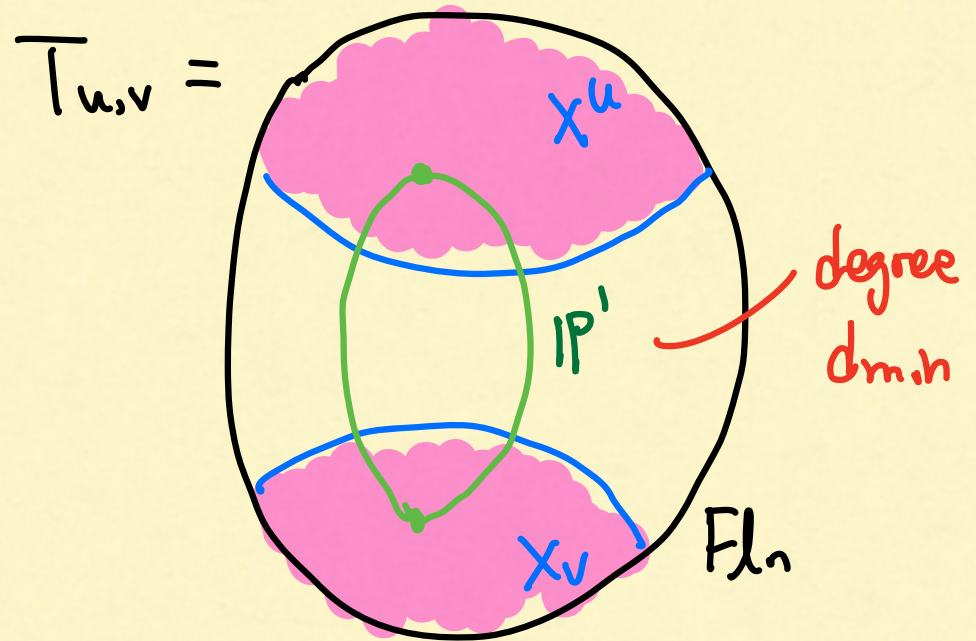
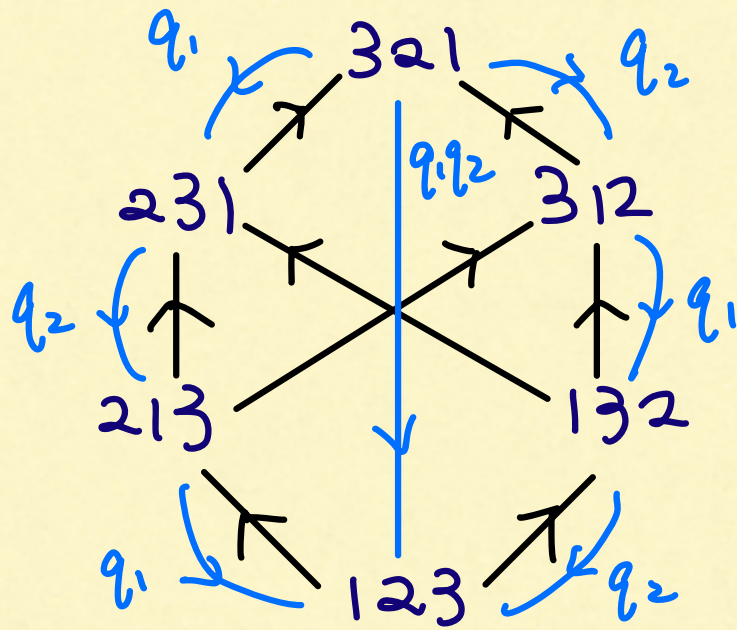
Tilted Richardson

Varieties

&

Jiyang (Johnny) Gao

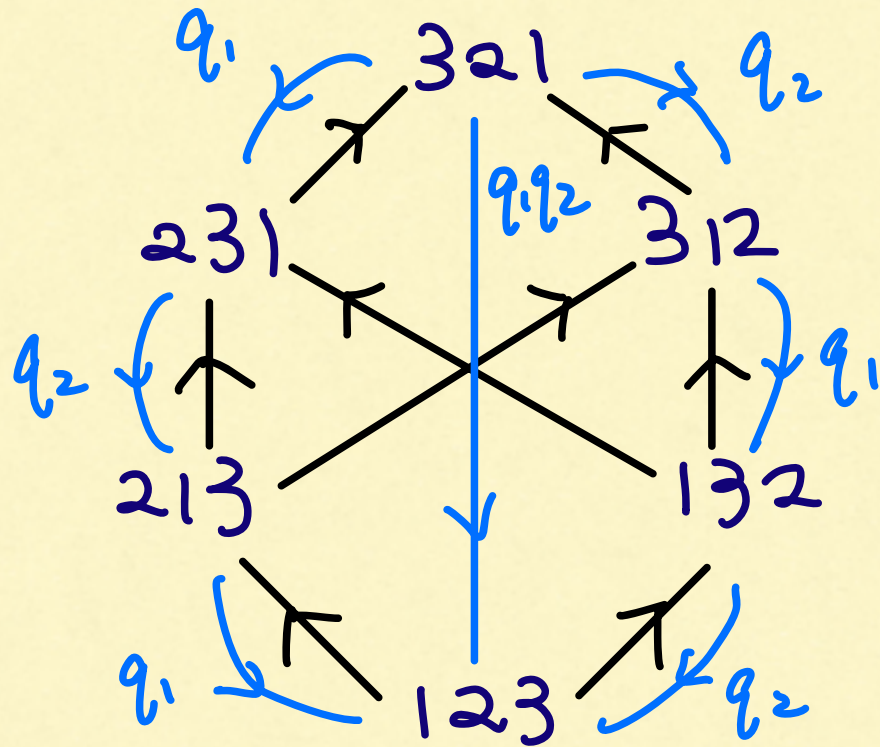
joint w/ Shiliang Gao, Yibo Gao*



* We are not related.

Quantum Bruhat Graph

[Brenti - Fomin - Postnikov '99]



Strong Bruhat Edges:

$$w \xrightarrow{1} wt_{ij}$$

$$\text{if } l(wt_{ij}) = l(w) + 1$$

Quantum Edges:

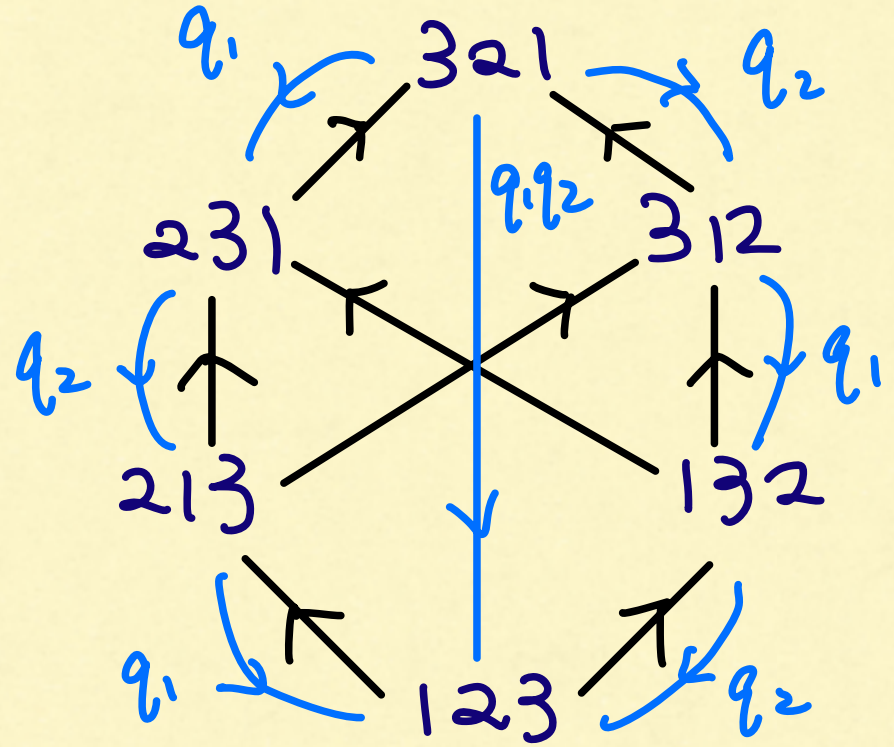
$$w \xrightarrow{q_i q_{i+1} \cdots q_{j-1}} wt_{ij}$$

$$\text{if } l(wt_{ij}) = l(w) - l(t_{ij})$$

Properties of Quantum Bruhat Graphs

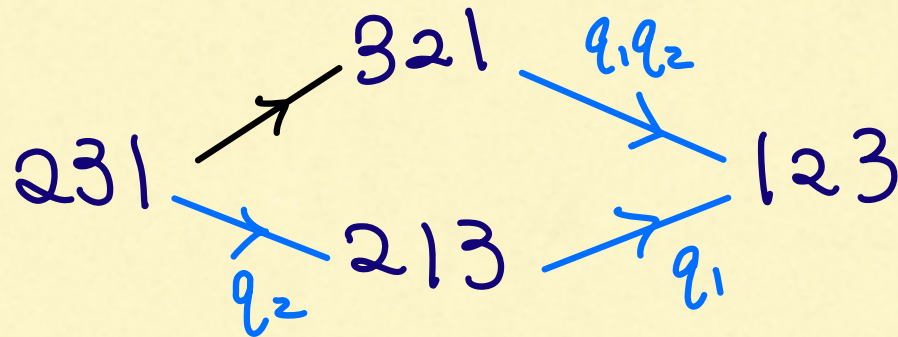
Thm [Postnikov '04]

All shortest paths $u \rightarrow v$
have same weight d_{\min}



Ex: Quantum Bruhat Interval

$[231, 123] :=$

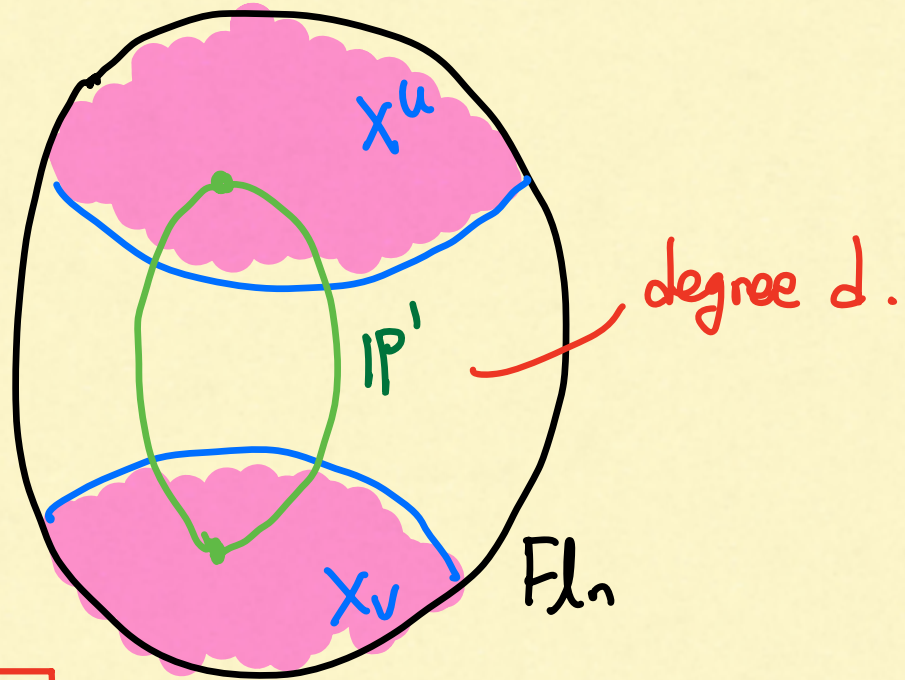


Weight d_{\min}
$q_1 q_2$

Curve Neighbourhood

[Buch - Chaput - Mihalcea - Perrin '13]

$$\Gamma_d(X^u, X_v) := \bigcup_{P' \in \text{Fl}_n} \dots$$



$$\overline{\text{Fl}}_n = \text{Mat}_{n \times n} \square / B$$

B
 $\begin{matrix} * & * & * & * \\ & * & * & * \\ & & * & * \\ & & & * \end{matrix}$

Schubert
Variety

$$X_{3142} = \left\{ \begin{matrix} 3 & 1 & 4 & 2 \\ * & 1 & 0 & 0 \\ * & 0 & * & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{matrix} / B \right\}$$

When $d = d_{\min}$,

$T_{u,v} := \Gamma_{d_{\min}}(X^u, X_v)$ is

the tilted Richardson Variety

Main Result [Gao³ 24'+]

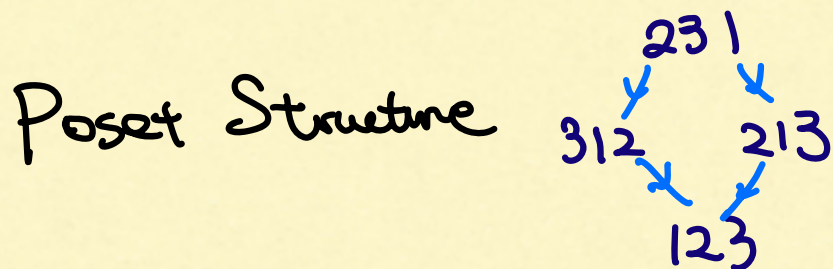
Combinatorics

Quantum Bruhat Interval $[u, v]$

Length $l([u, v])$

Containment $[x, y] \subseteq [u, v]$

Element $w \in [u, v]$



Geometry

Tilted Richardson Variety $T_{u, v}$

Dimension $\dim(T_{u, v})$

Stratification $T_{x, y} \subseteq T_{u, v}$

Permutation matrix $\begin{bmatrix} & \omega & \\ & & \\ & & \end{bmatrix} \in T_{u, v}$
(T -fixed points)

Face poset of $T_{u, v}^{\geq 0}$ *

~~More~~ on tilted Richardson Varieties

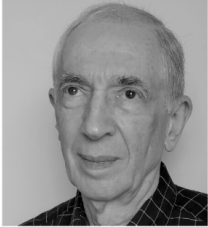
Jokes

See y'all here ↗

Harvard and MIT Present

The Many Combinatorial Legacies of Richard P. Stanley

Immense Birthday Glory of the Epic Catalanian Rascal




When
June 3-7, 2024

Location
Harvard Geological Lecture Hall

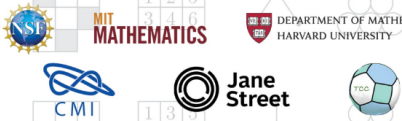
Speakers

George Andrews	Soichi Okada
Sara Billey	Igor Pak
Anders Björner	Jim Propp
Persi Diaconis	Alan Sokal
Sergey Fomin	Kenneth Stanley
Phil Hanlon	Sharon Stanley
Takayuki Hibi	Michelle Wachs
Donald Knuth	Stephanie van Willigenburg
Christian Krattenthaler	Peter Winkler
Isabella Novik	Catherine Yan

Scan for more information and to register:



Organizers: Alejandro Morales (UQAM, UMass Amherst), Tom Roby (UConn) and Lauren Williams (Harvard University)
MIT contacts: Alex Postnikov and Yufei Zhao



Sponsored by the National Science Foundation, MIT Mathematics, Harvard University Mathematics, MIT Richard P. Stanley Fund in Combinatorics, Clay Mathematics Institute, Jane Street, and The Combinatorics Consortium.
Poster graphics courtesy of Alejandro Morales.